SINTERING OF POWDERS AND DENSE MATERIALS
Experimental, Mechanical, Thermal approaches and Modelling

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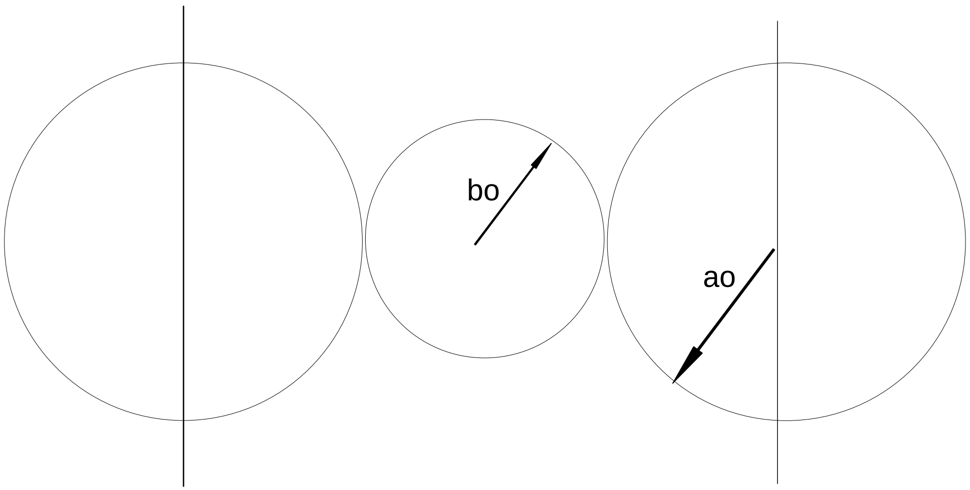
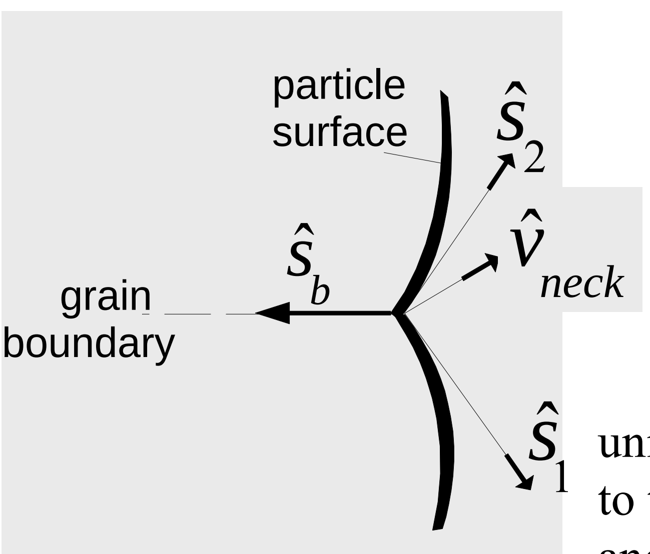
I
SINTERING OF POWDERS AND DENSE MATERIALS
Experimental, Mechanical, Thermal approaches

Pr Ange Nzihou

Geometry of the problem

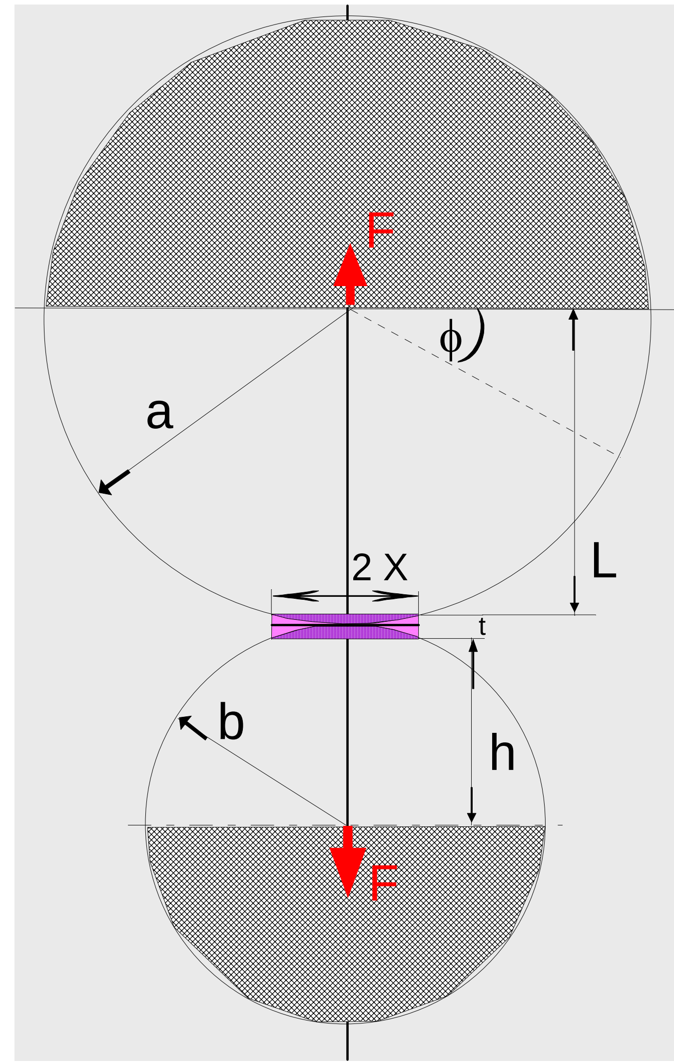


a row of spherical particles in contact



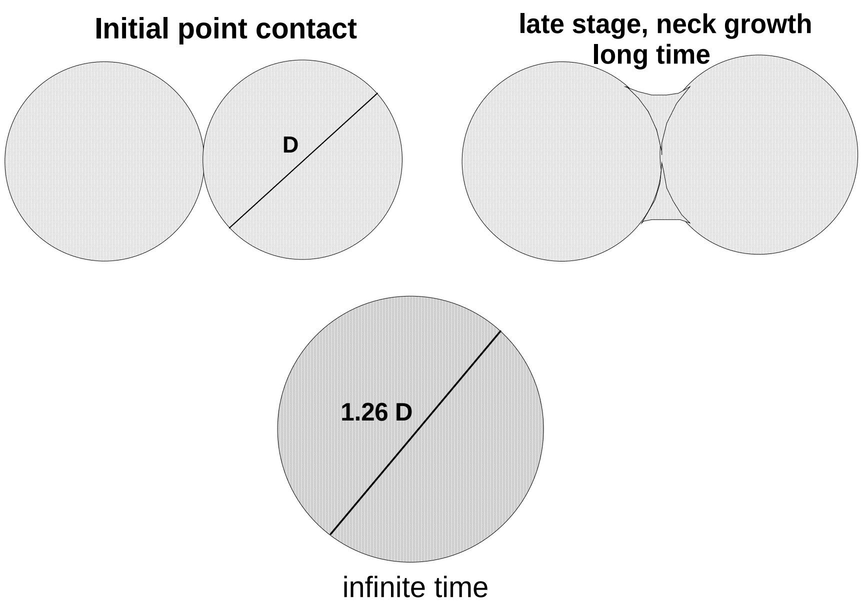
velocity of the necks

unit tangent vectors to the grain boundary and particle surface at the necks

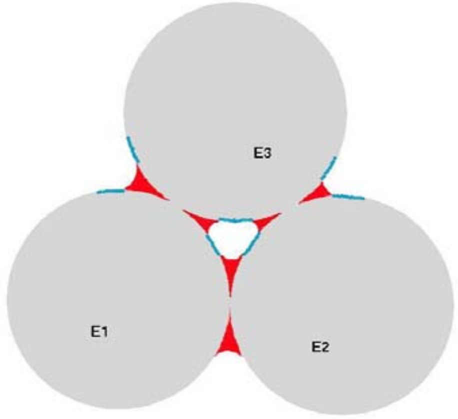


unit problem representing the geometry during neck formation

SINTERING



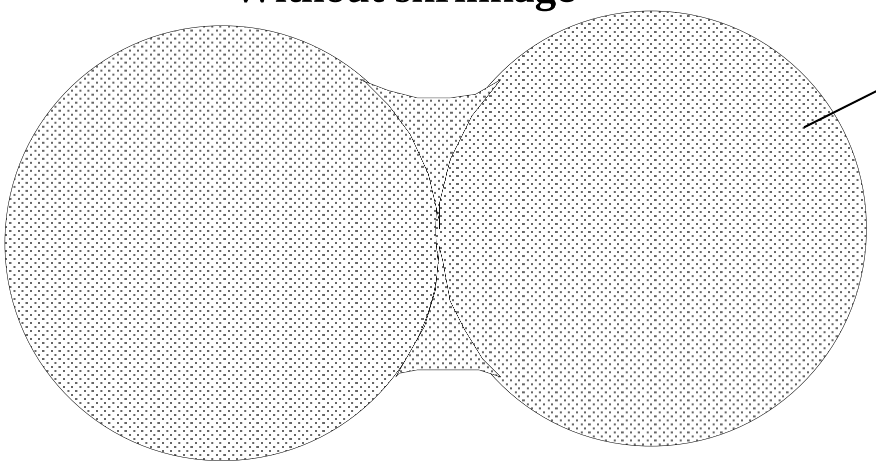
Sintering model with the development of the particle bond during sintering



Microstructural scale
In boundary, important atomic motion

MACROSCOPIC DESCRIPTION

Without shrinkage

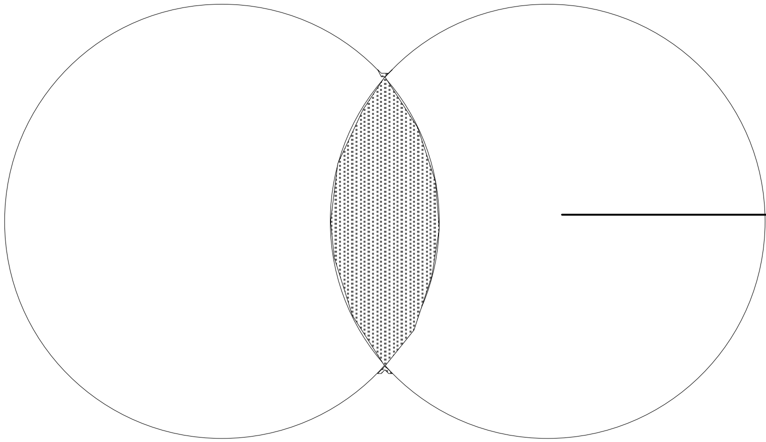


Principal mechanisms

Evaporation –condensation
Superficial diffusion

- Formation of pores
- Mechanical resistance
- Chemical reactions
- Dimensional variation contingent on temperature

With shrinkage



Principal mechanisms

Volume diffusion
flows (viscous and plastic)

PHYSICAL CHEMISTRY ASPECT

DRIVING ENERGIES:

Surface Energy :

•Surface tension $\gamma = dW/dS$

Energy linked to the presence of physical defects :

•Proximity of curved surfaces

•In the crystalline network, there exists a concentration of C de lacuna expressed as flows(thermodynamic statistics):

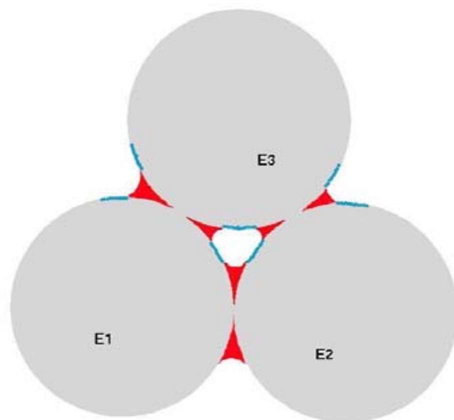
$$C_o = \frac{n}{N} \approx \exp(E_f/kT)$$

Energy linked to the presence of pressure :

• If the interface is curved, the pressure of the vapor, in equilibrium with the solid, changes depending on the curvature of the surface

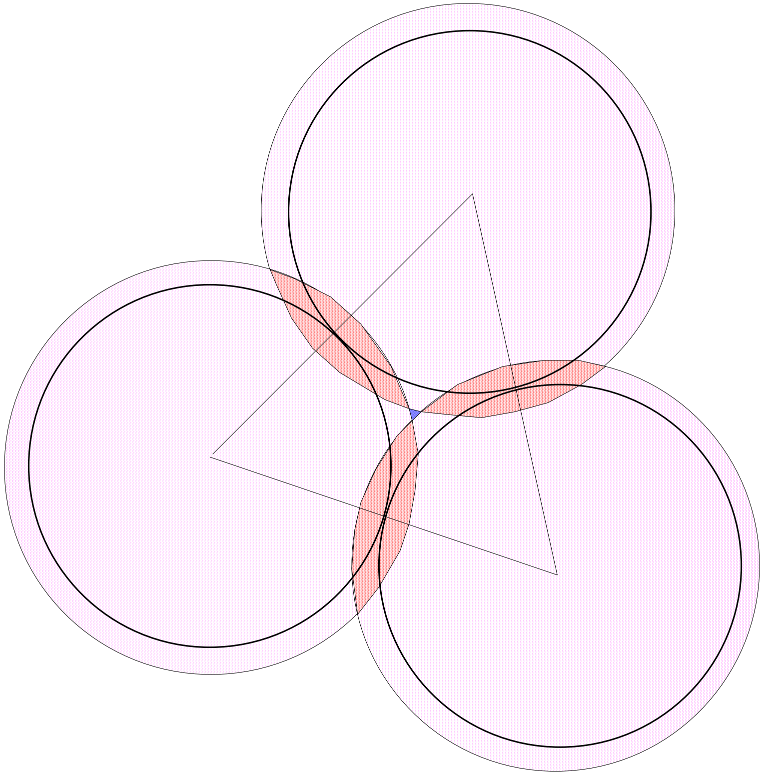
SINTERING MECHANISMS IN SOLID PHASE

1-TANGENT SPHERES



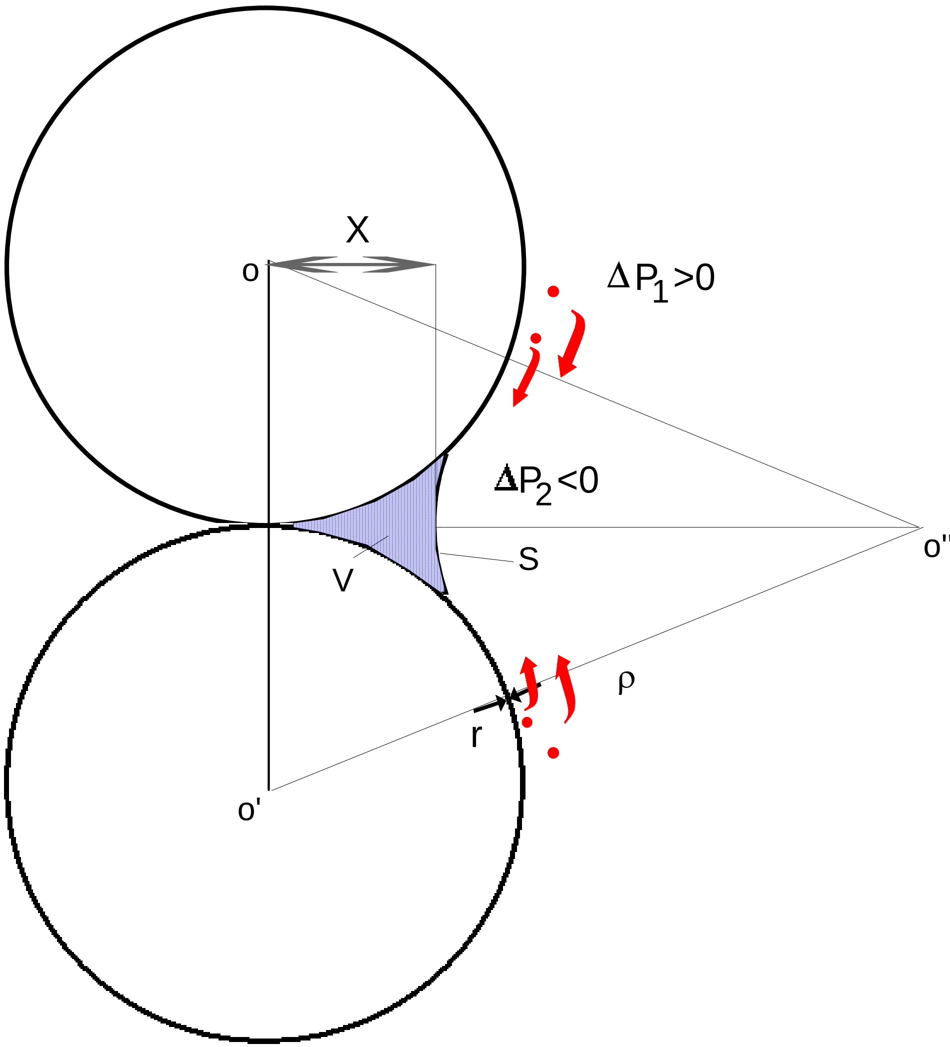
1. Evaporation –condensation
2. Superficial diffusion
3. Volume diffusion

2- SECANT SPHERES



1. Viscous flow
2. Volume diffusion
3. Intergranular diffusion
4. Microcreep

1.1-TANGENT SPHERES: Evaporation - condensation



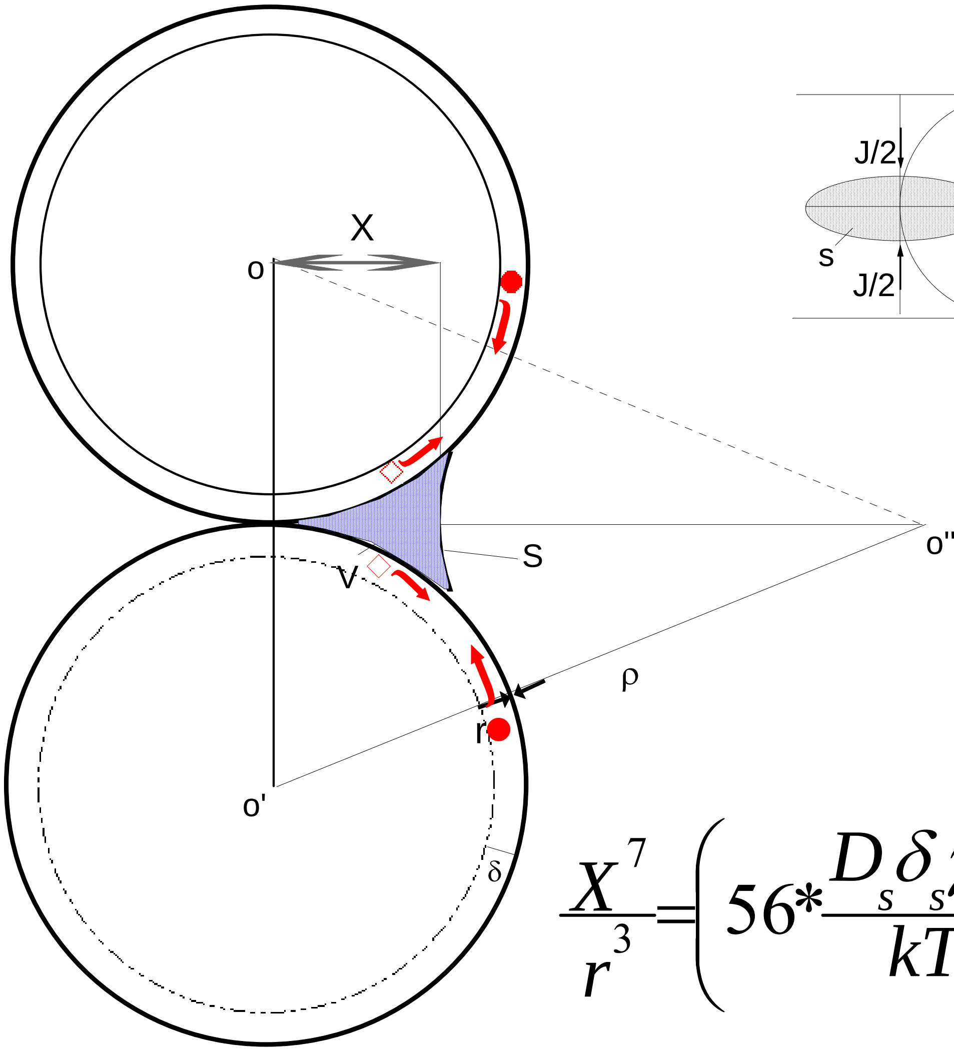
- That a transfer of atoms will be established, by the gaseous phase from the sphere's surface toward the lateral surface of the bridge.

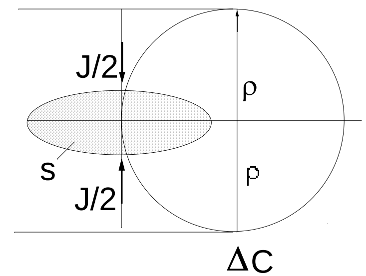
$$\Delta P_1 > 0 \quad et \quad \Delta P_2 < 0$$

$$\frac{X^3}{r} = \left[\frac{3\pi\gamma P_0 \Omega}{dkT} * \left(\frac{M}{2\pi RT} \right)^{1/2} \right] * t$$

$\Omega = volume \quad atomique$

1.2-TANGENT SPHERES : superficial diffusion





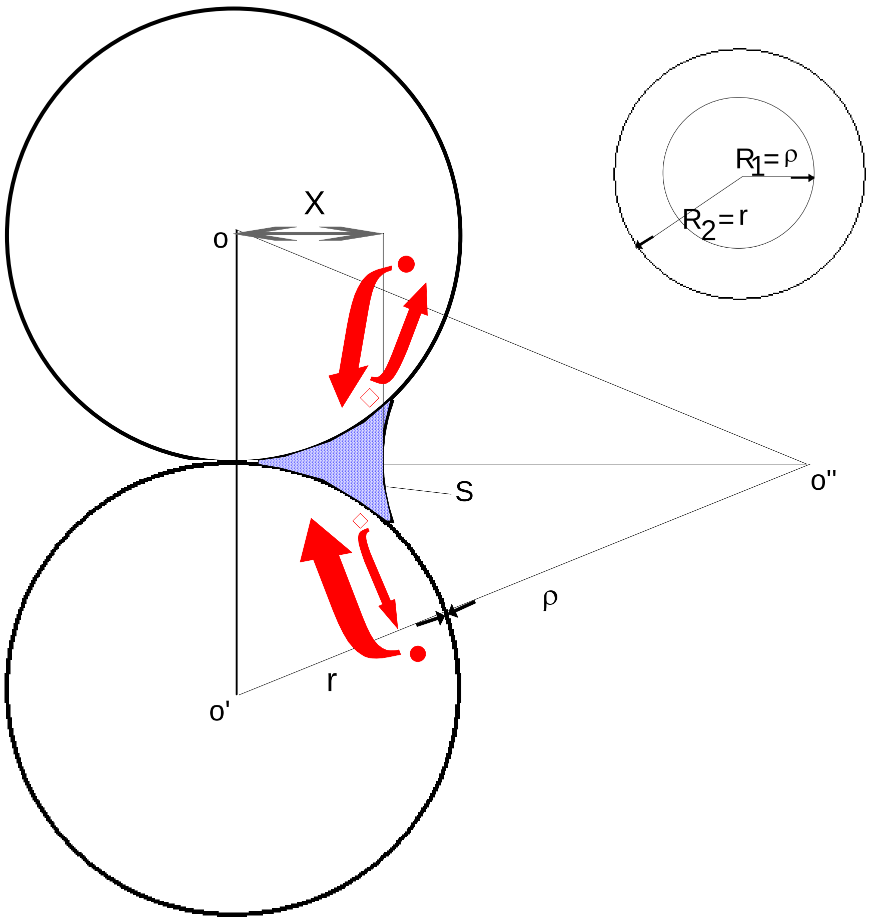
•In proximity to the bridge's surface, there exists and excess of lacunas; however, Nearby the sphere's surface –far from the threshold– there exists a defect of lacunas.

•The extra lacunas will diffuse. If a flux of lacunas is established, there will be an equivalent flux of atoms in the opposite direction which will therefore contribute to build up the bridge.

$$\frac{X^7}{r^3} = \left(56 * \frac{D_s \delta_s \gamma \Omega}{kT} \right) * t$$

This exchange of lacunas and atoms will only be restricted to the superficial layer δ_s

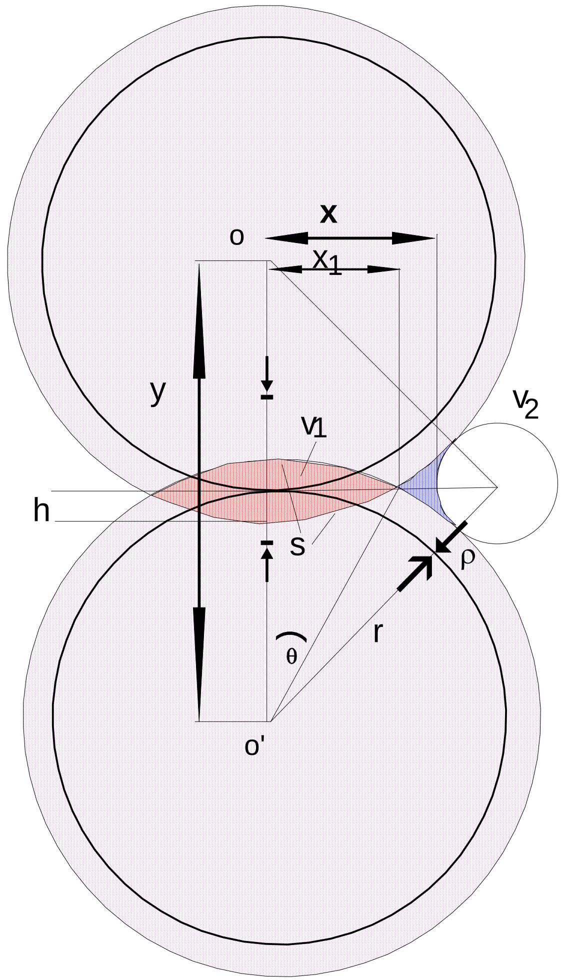
1.3-TANGENT SPHERES : Volume diffusion



- Based on the presence of an excess of lacunas neighboring the bridge's surface, and of a defect of lacunas nearby the sphere surfaces far from the bridge.
- This diffusion of lacunas (and of atoms) is speculated to operate on the volume and no longer on the surface

$$\frac{X^5}{r^2} = \left(\frac{5\pi}{2} * \frac{D_v \gamma \Omega}{kT} \right) * t$$

2.1-SECANT SPHERES : Viscous flow



- The formation of a linking zone between spheres carried out by viscous flow of Newtonian-type material
- the displacement of atoms carried out under the influence of a cut which is proportional to the gradient of speeds

$$\sigma = \eta \frac{d\varepsilon}{dt} \qquad \eta = viscosité \qquad \varepsilon = \frac{dl}{l}$$

$$\left(\frac{x^2}{r}\right)^{1/n} = K\left(\frac{n\gamma}{\eta}\right)^{1/n} * t$$

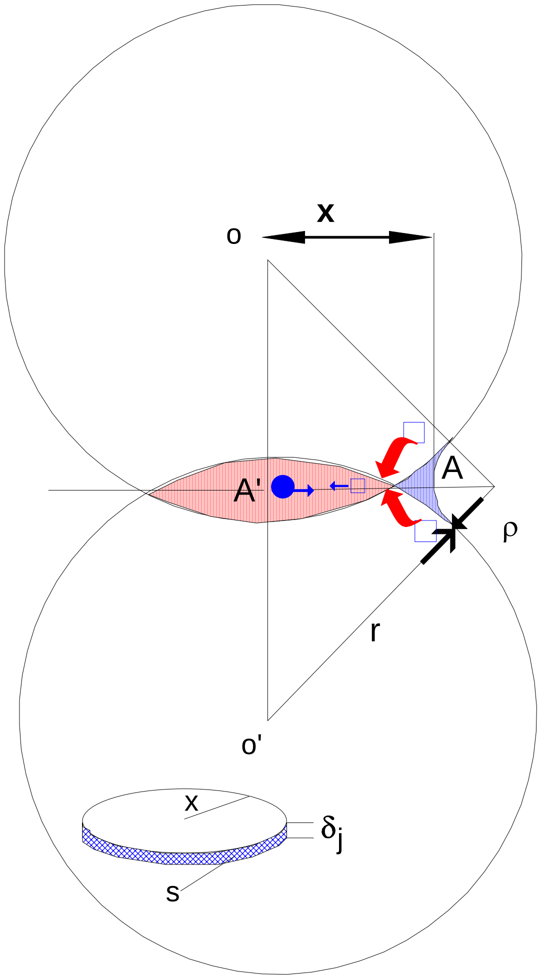
2.2-SECANT SPHERES : Volume diffusion

- Similar to those previously mentioned.

$$\frac{x^5}{r^2} = \left(20\pi \frac{D_v \gamma \Omega}{kT}\right) * t$$

- The model considered a variation of the distance between the centers of the spheres

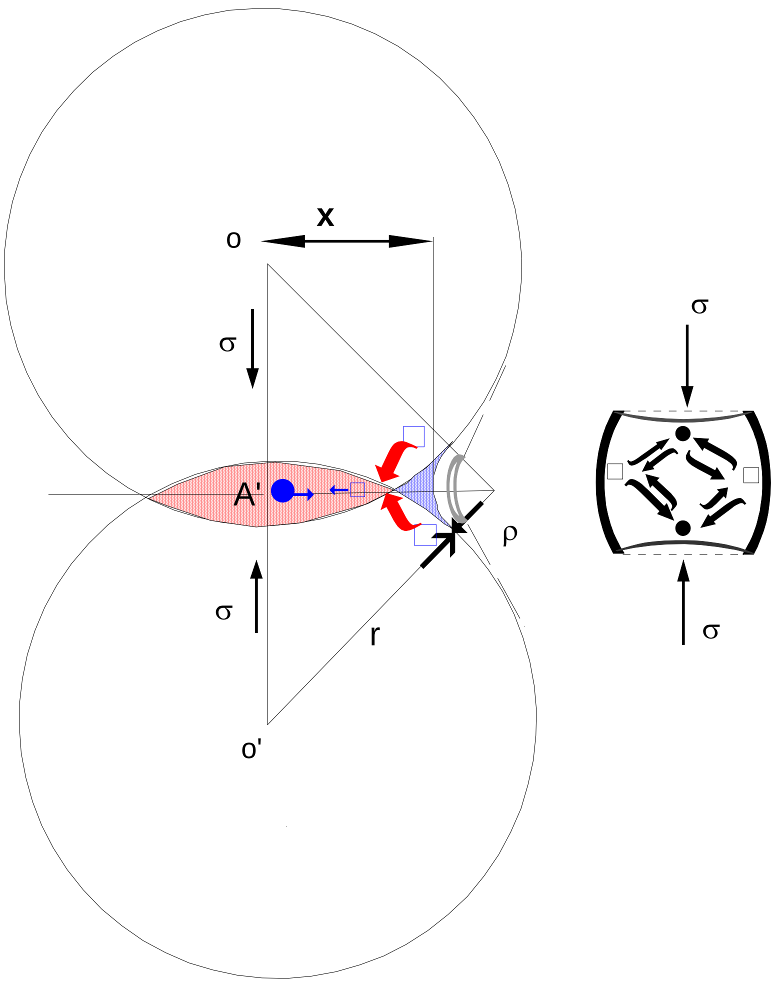
2.3-SECANT SPHERES : Intergranular diffusion



- The experimental observations show that in the majority of cases, it forms a neck within the linked zone, being AA'.
- Les lacunas, finding themselves in excess neighboring the concave surface of the bridge, will be able to diffuse toward this grain joint, instead of spreading to surfaces with a larger radius of curvature meaning the surfaces of the two spheres.

$$\frac{x^6}{r^2} = \left(96 \frac{D_j \delta_j \gamma \Omega}{kT} \right) * t$$

2.4- SECANT SPHERES : Microcreep mechanism

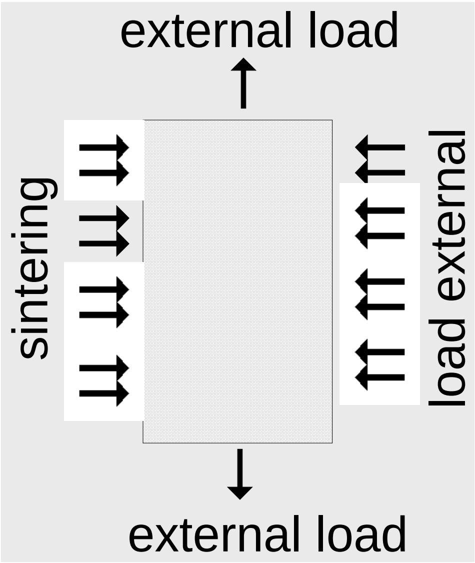


- Creep indicates a flow of material occurring at a given temperature under the influence of a constant.
- Volume subject to the strain of compression σ_1

$$\sigma \cong \gamma \left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2} \right)$$

Cases:

Sintering of a homogeneus body under uniaxial loading (tension or compression)



- Sinter forging
- Plastic flow and sintering under the action of external forces
  - overall compression
  - uniaxial loading
  - torsion

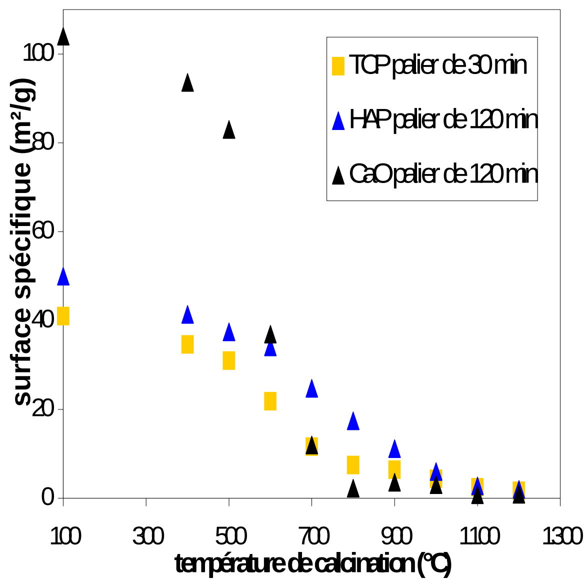
Free sintering

- free sintering of linear – viscous porous material
- power – law creep (nonlinearity of the constitutive properties)

Evolution des solides traités (contenant des polluants)

Typical evolution

Reduction of the specific surface area



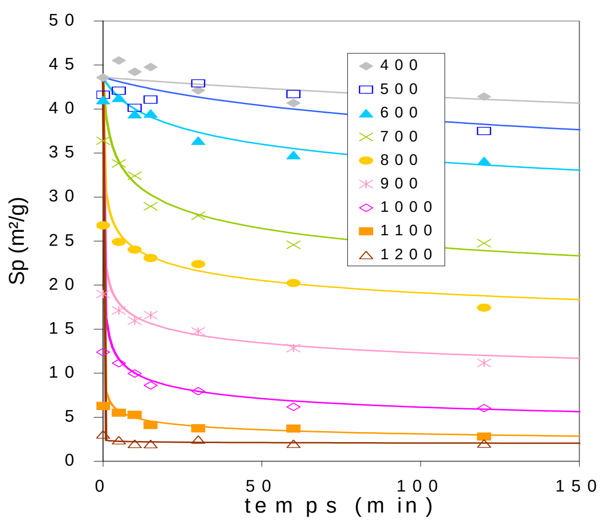
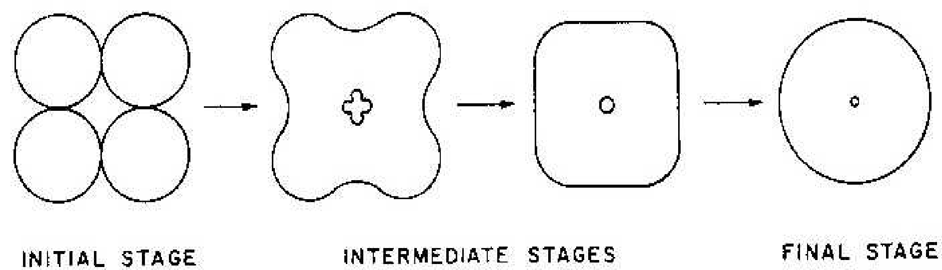
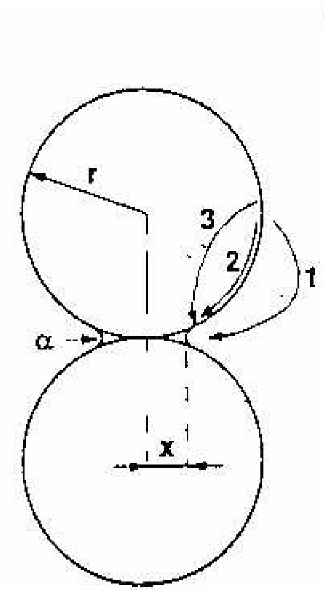


Illustration of the sintering phenomenon:





$$j_i = -\frac{D_i}{RT} \, grad \, \sigma , D_i = D_{0i} e^{-E_i / RT}$$

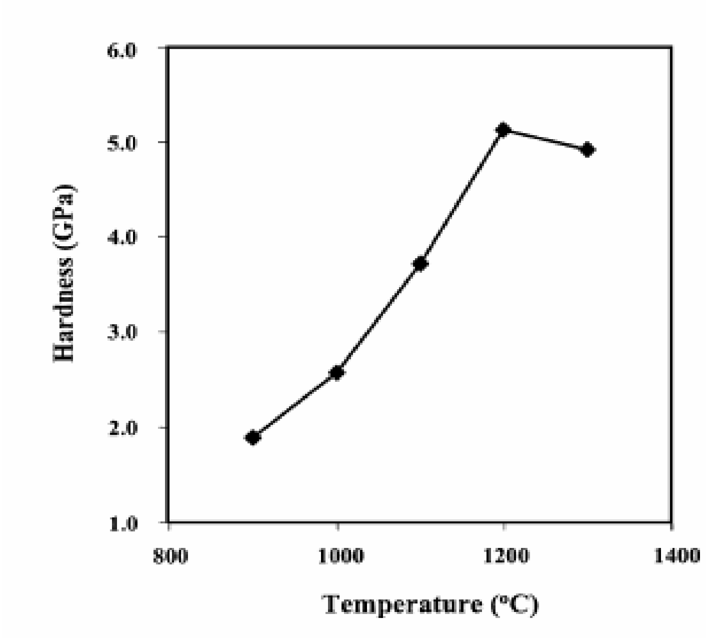
- **Diffusion in gas phase (1)**
- **Surface diffusion (2)**
- **Volume diffusion (3)**

Kinetics:

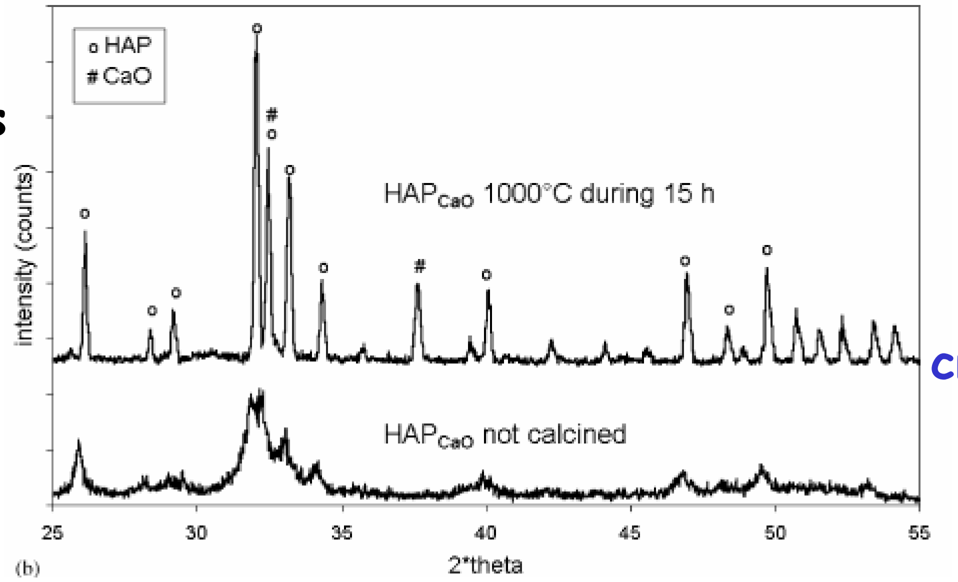
$$-\frac{dS}{dt} = kS^n$$

Effect of Calcination on HAp properties

Other physical changes



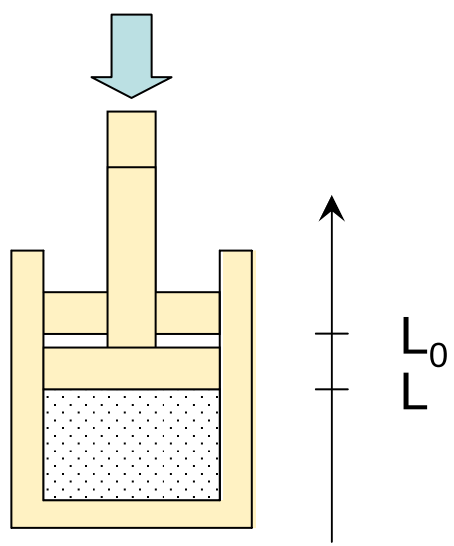
Chemical changes

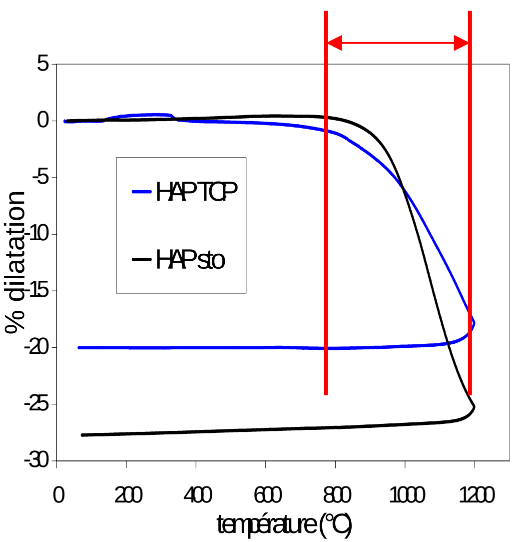


Amorphous
to
cristalline structure

Calculation of properties during the sintering

Density and porosity calculation from TMA, Thermomechanical Analyzer





Shrinkage

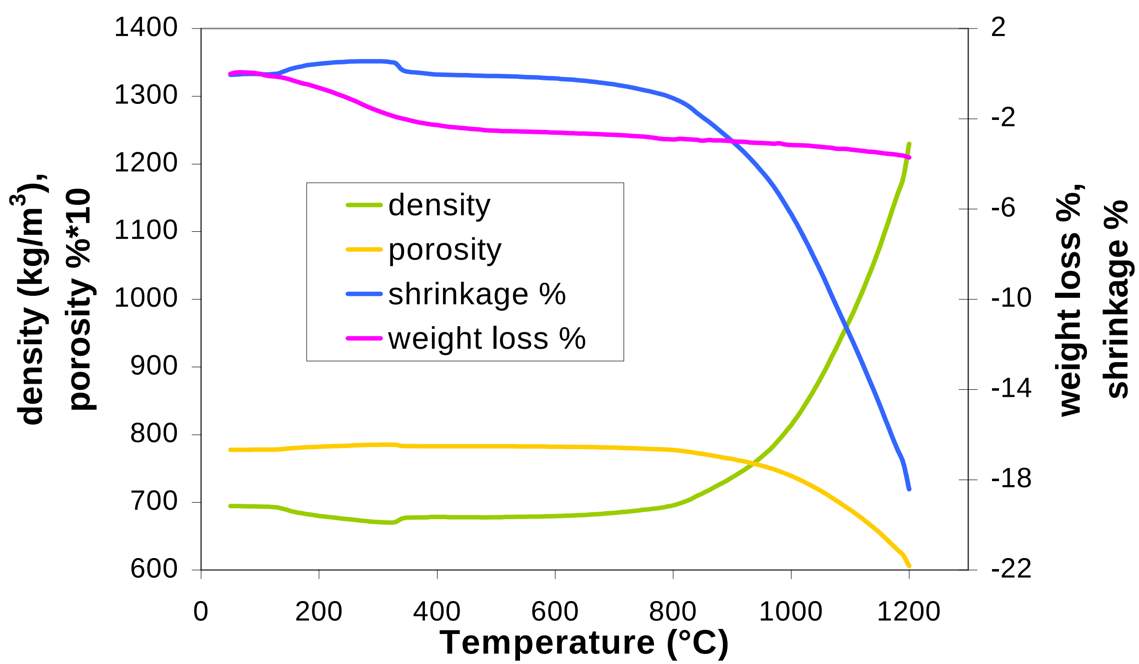
$$\frac{\Delta V}{V_0} = 1 - \left(\frac{L}{L_0}\right)^3$$

Density

$$\rho = \frac{m}{V} = \frac{m_0 - \Delta m}{V_0 - \Delta V}$$

$$\rho = \frac{m_0\left(1 - \frac{\%\,TG}{100}\right)}{\pi R_0^{\,2} h_0 \left(1 + \frac{\%\text{dilatation}}{100}\right)^3}$$

Porosity: $\varepsilon = 1 - \frac{\rho}{\rho_{theo}}$



Modeling for the description of the sintering process

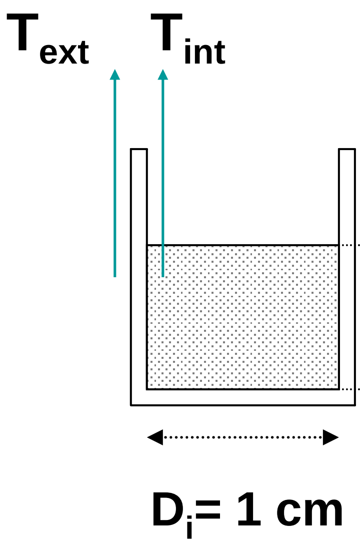
Typical representation of processes involved:

$$\rho(T,t)c_p(T)\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda r^2 \frac{\partial T}{\partial r}\right) + \nu \Delta H_r$$

ν: Kinetic of reaction

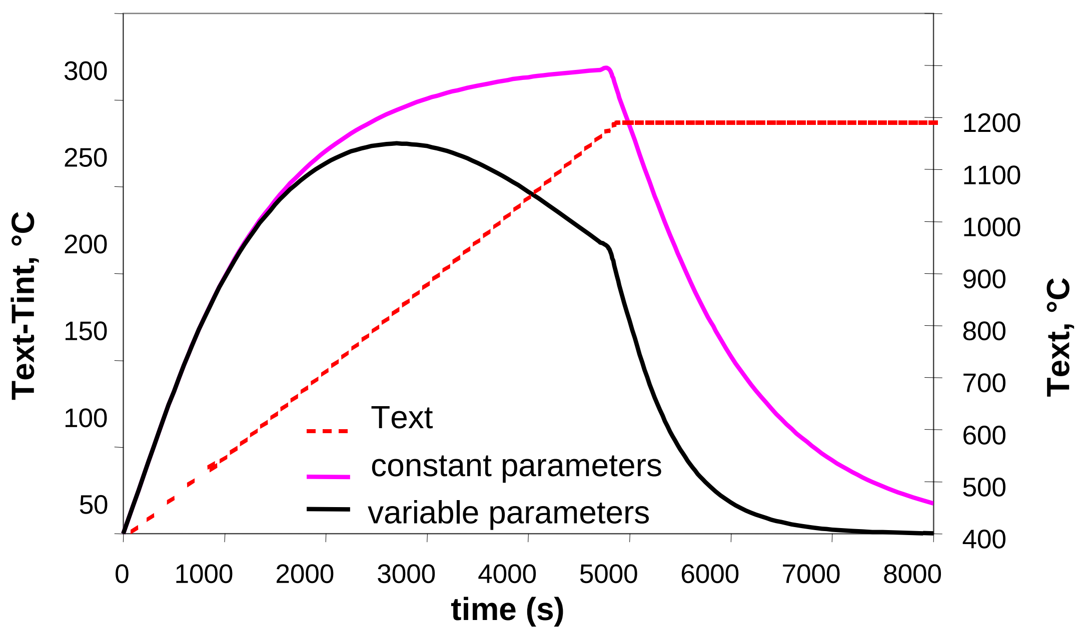
Continious measurement of the properties:

$$\alpha = \frac{\lambda(T,t)}{\rho(T,t)C_p(T,t)}$$



m_{HAP}= 2g

H= 2 cm



Better description of the sintering phenomenon

I
SINTERING OF POWDERS AND DENSE MATERIALS MODELLING

Pr Roberto Santander

Model for Sintering and Coarsening of Rows of Spherical Particles

Literature :

- Parhami F. et al; Mechanics of Materials – 31 pp 43-61 (1999)
- Svoboda and Riedel; Acta Mettal. Mater – Vol 43 N 1pp 1-10 (1995)
- Olevsky E; Materials Science and Engineering R23 (1998)

Goal :

Model for the formation of interparticle contacts and neck growth between powder particles by grain boundary and surface diffusion.

Methodology :

Model is based on a Thermodynamic Variational Principle arising from the governing equations of mass transport on the free surface and grain boundaries

↓

Maximization of the rate of dissipation of Gibbs free energy (for a pair of particles through grain boundary and surface diffusion)

Phenomenological model of sintering based upon the ideas of thermodynamics of irreversible processes

1. Porous medium is considered as a two phase material (phase of substance – body skleleton – and phase voids –pores)
2. The skleton is assumed to be made of individual particles having nonlinear – viscous incompresible isotropic behavior.
3. The voids (pores) are isotropically distributed.
4. The overall response is therefore isotropic
5. The free energy F per unit mass of porous medium is by hypothesis, a function of the absolute temperature T and of the specific volume v.

<u>Second principle de la thermodynamique des milieux continus</u>

$$\rho\left(T\dot{s}-\dot{e}\right)+\overline{\overline{\sigma}}:\dot{\overline{\overline{\varepsilon}}}-\vec{q}\cdot\frac{\overrightarrow{grad}\ T}{T}\geq 0$$

internal energy — internal stress — heat rate

énergie libre spécifique F

$$F=e-Ts$$

Phenomenological model of sintering based upon the ideas ofthermodynamics of irreversible processes;

Phenomenological model of sintering based upon the ideas of thermodynamics of irreversible processes

Clausius' inequality – Duhem with hypothesis F=F(T,v) and

$\vec{grad}\ T=0$

σ → Cauchy stress tensor

ε → strain rate tensor

$$\sigma_{ij}:\dot{\varepsilon}_{ij}-\rho\dot{F}-\rho s\dot{T}\geq 0$$

$$\bar{\varepsilon}=\bar{\bar{\varepsilon}}^{e}+\bar{\bar{\varepsilon}}^{p}$$

Plastic distortion (irreversible)

Elastic distortion (reversible)

$$\sigma_{ij}=f(\varepsilon_{ij})$$

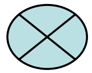
for a viscous material $S=f(\varepsilon_{ij})$ } but not on $\dot{T}$

$$S=-\frac{\partial F}{\partial T}$$

$$(\sigma_{ij}-P_L\delta_{ij})\dot{\varepsilon}_{ij}\geq 0$$

where $P_L=\left.\frac{\partial F}{\partial v}\right)_T$ Laplace pressure or sintering stress (result of the collective action of local capillary stresses in a porous material)

Phenomenological model of sintering based upon the ideas of thermodynamics of irreversible processes

The condition  Is satisfied if there exists a dissipative potential D defined as a homogeneous function of order m+1 of the strain rate $\dot{\varepsilon}_{ij}$

$$\sigma_{ij} - P_L \delta_{ij} = \frac{\partial D}{\partial \dot{\varepsilon}_{ij}} \quad \text{and} \quad \dot{\varepsilon}_{ij} \frac{\partial D}{\partial \dot{\varepsilon}_{ij}} = (m+1) D \geq 0$$

$$D = f(v) \quad or \quad D = f(\theta) \qquad \theta = \text{porosity} = \frac{V_{pores}}{V_{total}}$$

For D, cases three

1. Linear incompressible viscous material with voids
2. Incompressible nonlinear – viscous material
3. Nonlinear – viscous porous material

Phenomenological model of sintering based upon the ideas of thermodynamics of irreversible processes

1. Linear viscous incompressible material with voids

dissipation potential $D = \eta\gamma^2 + \frac{1}{2}\zeta e^2$ → shrinkage rate

$\gamma = \sqrt{\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}}$ → deviator of the strain rate tensor

shear modulus of the porous body skeleton

effective shear module

effective bulk module (viscosity for example)

$e = tr\,\dot{\varepsilon} = \dot{\varepsilon}_{ii} = -\frac{\dot{\rho}}{\rho}$

$\varphi = (1-\theta)^2$

$$\eta = \varphi\eta_0 \qquad \zeta = 2\psi\eta_0 \qquad \psi = \frac{2}{3}\frac{(1-\theta)^3}{\theta}$$

the potential D can be expressed as: $D = (1-\theta)\,\eta_0 W^2$

$$W = \sqrt{\frac{\varphi\gamma^2 + \psi e^2}{1-\theta}}$$

The constitutive law is:

$$\sigma_{ij} = 2\eta_0(\varphi\dot{\varepsilon}'_{ij} + \psi e\,\delta_{ij}) + P_L\delta_{ij}$$

Phenomenological model of sintering based upon the ideas of thermodynamics of irreversible processes

2. Incompressible nonlinear – viscous material

- incompressible material ⟶ $\theta = 0$ ⇨ $\varphi \to 1 \quad and \quad \psi \to \infty$
- incompressible matric ⟶ $e \to 0$ ⇨ $\psi e^2 \to 0 \quad therefore \quad W \to \gamma$
- the dissipative potential of a linear – viscous fluid is $D = \eta_0 \gamma^2$
- an extension into nonlinear –viscous behavior is obtained by

material parameter depends on temperature (→ A)

strain rate sensitivity (→ m)

$$D = \frac{A}{m+1}\gamma^{m+1}$$

- the deviatoric stress is obtained from

$$\sigma'_{ij} = \frac{\partial D}{\partial \dot{\varepsilon}_{ij}} = A\gamma^{m+1}\dot{\varepsilon}_{ij} \qquad (\dot{\varepsilon}_{ij} = \dot{\varepsilon}'_{ij})$$

Phenomenological model of sintering based upon the ideas of thermodynamics of irreversible processes;

3. Nonlinear – viscous porous material

- using a power law dependence, the dissipation of a porous material is

$$D=\frac{A}{m+1}(1-\theta)W^{m+1}$$

- matrix incompressible, nonlinear-viscous

$$D_{matrix}=\frac{A}{m+1}\gamma^{m+1}$$

$\theta=0$ we have $W=\gamma$

(incompressible nonlinear viscous material)

$m=1$ we have $A=2\eta_0$

(linear viscous porous material)

- constitutive law

$$\sigma_{ij}=AW^{m-1}(\varphi\dot{\varepsilon}'_{ii}+\psi e\delta_{ij})+P_L\delta_{ij}$$

if

$\tau=A\varphi W^{m-1}\gamma$ $p=A\psi W^{m-1}e+P_L$

substituting leads to the following relationship between the equivalent stress σ and the equivalent strain rate W

$$\sigma=AW^m$$

Phenomenological model of sintering based upon the ideas of thermodynamics of irreversible processes;

- a generalization of the relationship, $\sigma = A\,W^{m}$ between equivalent stress and equivalent strain rate $\sigma = \sigma(W)$ ⟶ arbitrary function of W

- in this general case, the constitutive relationship for a nonlinear –viscous porous material can be represented in the form (sintering under pressure):

$$\sigma_{ij} = \frac{\sigma(W)}{W}\left(\varphi \dot{\varepsilon}'_{ii} + \psi\, \dot{e}\, \delta_{ij}\right) + P_L \delta_{ij}$$

« basic constitutive expression for the continuum theory of sintering »

- σ_{ij}: externally applied stresses
- $\frac{\sigma(W)}{W}(\varphi \dot{\varepsilon}'_{ii} + \psi\, \dot{e}\, \delta_{ij})$: material resistance
- $P_L \delta_{ij}$: influence of capillary stresses (sintering factor)

if :

- $\sigma_{ij} = 0 \rightarrow$ free sintering
- $P_L = 0 \rightarrow$ treatement by pressure without sintering
- $\sigma_{ij} \neq 0$ and $P_L \neq 0 \rightarrow$ sintering under pressure

Phenomenological model of sintering based upon the ideas of thermodynamics of irreversible processes

we have

$$\tau = \frac{\sigma(W)}{W}\varphi\gamma \quad \text{and} \quad p = \frac{\sigma(W)}{W}\psi e + P_L$$

- an important relationship between the invariants of stress – strain rate state is:

$$(p - P_L)\varphi\gamma = \tau\psi e$$

Model Formulation

• At hight temperature, atoms travel along the free surfaces and the interparticle contacts to reduce the total free energy of surfaces and interfaces of the system.

<u>ENERGY BALANCE BETWEEN SOURCES AND SINKS</u>

$$\Pi = \dot{G}_s + R_s$$

G_s : rate of change of the free energy of system

R_s : one-half of the rate of energy dissipation

$$\dot{G}_s = \gamma_s \dot{A}_s + \gamma_b \dot{A}_b - Fv$$

$$R_s = \frac{1}{2}\int_{A_b} \frac{1}{D_b} \vec{J}_b \cdot \vec{J}_b \, dA_b + \frac{1}{2}\int_{A_s} \frac{1}{D_s} \vec{J}_s \cdot \vec{J}_s \, dA_s$$

$\gamma_s \wedge \gamma_b \rightarrow$ surface and grain boundary energies per unit area

$$D_s = \frac{\delta_s \vec{D}_s \Omega}{kT} \qquad D_b = \frac{\delta_b \vec{D}_b \Omega}{kT}$$

$A_s \wedge A_b \rightarrow$ surface and grain boundary areas

$F \rightarrow$ applied force

$v \rightarrow$ velocity of one end of the row of particles to the other

$D_b \wedge D_s \rightarrow$ diffusion parameters

$\vec{J}_b \wedge \vec{J}_s \rightarrow$ fluxes of material on the grain boundary and on the free surface

$\delta_b \wedge \delta_s \rightarrow$ thicknesses within wich diffusion occurs on the grain boundary and surfaces

$\vec{D}_b \wedge \vec{D}_s \rightarrow$ grain boundary and surface atom diffusivities

$\Omega \rightarrow$ atomic volume

$k \rightarrow$ Boltzman's constant

$T \rightarrow$ absolute temperature

Model Formulation

Important

$J \rightarrow$ volume of material passing by diffusion through unit length in unit time

$\dot{G}_s \rightarrow$ the rate of change of the free energy of the system is the rate of change of the internal energy minus the external work rate. In the system, internal energy is the sum of surface and grain boundary energies

$\prod \rightarrow$ has a stationary minimum value with respect to compatible variations of J_b, J_s, A_s, A_b and v

The Rayleigh - Ritz minimization is achieved by setting

$$\delta \prod = \delta \dot{G}_s + \delta R_s = 0$$

δΠ must be zero for variations

$$\left\{ \begin{array}{l} \delta \dot{A} \\ \delta \dot{b} \\ \delta \dot{L} \\ \delta \dot{h} \\ \delta \dot{t} \\ \delta \dot{x} \end{array} \right\}$$

degrees of freedom of the Variational functional Π

Model Formulation ………………. compatibility condition

sum of the principal curvatures of the particle surface

$$\dot{A}_s = \int_{A_s} \kappa v_n \, dA_s + \sum_{\text{all b}} \int_{neck} v_{neck} \cdot (\hat{s}_1 + \hat{s}_2) dL$$

motion of the locus of point of connection between the grain boundary and free surface

rate of motion of the particle surface in the outward normal direction

$$\dot{A}_b = \sum_{\text{all b}} v_{neck} \cdot \hat{s}_b \, dL$$

$$Fv = \sum_{\text{all b}} \int_{A_b} \sigma \, dA_b v_b$$

integral of the strees over each grain boundary

Equations of conservation

$$v_n + \nabla_s \cdot \vec{J}_s = 0$$ → volume conservation on A_s

$$v_b + \nabla_b \cdot \vec{J}_b = 0$$ → volume conservation on grain boundary

$$\hat{s}_1 \cdot \vec{J}_{s1} + \hat{s}_2 \cdot \vec{J}_{s2} + \hat{s}_b \cdot \vec{J}_b = 0$$ → flux continuity at the necks requires

$$\gamma_s \kappa_1 = \gamma_s \kappa_2 = -\sigma \quad \text{on L}$$ → continuity of chemical potential

$$J_s = -D_s \gamma_s \nabla_s \kappa \quad \text{on } A_s$$

$$J_b = D_b \nabla_b \sigma \quad \text{on } A_b$$

Model Formulation

The resulting expressions are coupled linear equations :

$$\left[\hat{k}\right]\left\{\dot{\delta}\right\}=\left\{\hat{f}\right\}$$

rates of change of the six degrees of freedom

$$\begin{bmatrix} \frac{\pi a^2}{xD_s}(2\,x\,a^2 g_a+L^2 t) & 0 & 0 & 0 & 0 & \frac{\pi a L t^2}{4D_s} \\ 0 & \frac{\pi b^2}{xD_s}(2xb^2 g_b+h^2 t) & 0 & 0 & 0 & \frac{\pi b h t^2}{4D_s} \\ 0 & 0 & \frac{\pi x^4}{8D_b} & \frac{\pi x^4}{8D_b} & \frac{\pi x^4}{8D_b} & 0 \\ 0 & 0 & \frac{\pi x^4}{8D_b} & \frac{\pi x^4}{8D_b} & \frac{\pi x^4}{8D_b} & 0 \\ 0 & 0 & \frac{\pi x^4}{8D_b} & \frac{\pi x^4}{8D_b} & \frac{\pi x^4}{8D_b} & 0 \\ \frac{\pi a L t^2}{4D_s} & \frac{\pi b h t^2}{4D_s} & 0 & 0 & 0 & \frac{\pi x t^3}{6D_s} \end{bmatrix} \begin{Bmatrix} \dot{a} \\ \dot{b} \\ \dot{L} \\ \dot{h} \\ \dot{t} \\ \dot{x} \end{Bmatrix} = \begin{bmatrix} -2\pi\gamma_s L \\ -2\pi\gamma_s h \\ -2\pi\gamma_s a+F \\ -2\pi\gamma_s b+F \\ -2\pi\gamma_s x+F \\ -2\pi\gamma_s t-2\pi\gamma_b x \end{bmatrix}$$

the set of variables used above are not independent, by imposing volume conservation and geometrical constraints, $L^2=a^2-x^2$ and $h^2=b^2-x^2$

Model Formulation

$$[k]\,\{\dot{\delta}\} = \{f\}$$

↳ rates of change of the three degrees of freedom

$$\begin{bmatrix} a^2 L^2\left(\frac{t}{x\,D_s}+\frac{1}{2\,D_b}\right)+\frac{2\,a^4\,g_a}{D_s} & \frac{a\,L\,b\,h}{2\,D_b} & a\,L\,x\,t\left(\frac{t}{4\,x\,D_s}+\frac{1}{2\,D_b}\right) \\ \frac{a\,L\,b\,h}{2\,D_b} & b^2 h^2\left(\frac{t}{x\,D_s}+\frac{1}{2\,D_b}\right)+\frac{2\,b^4\,g_b}{D_s} & b\,h\,x\,t\left(\frac{t}{4\,x\,D_s}+\frac{1}{2\,D_b}\right) \\ a\,L\,x\,t\left(\frac{t}{4\,x\,D_s}+\frac{1}{2\,D_b}\right) & b\,h\,x\,t\left(\frac{t}{4\,x\,D_s}+\frac{1}{2\,D_b}\right) & \frac{t^2\,x^2}{2}\left(\frac{t}{3\,x\,D_s}+\frac{1}{D_b}\right) \end{bmatrix} \begin{Bmatrix} \dot{a} \\ \dot{b} \\ \dot{x} \end{Bmatrix} = \begin{bmatrix} \frac{4\,\gamma_s\,a\,L}{x}\left(1-\frac{x}{2a}-\frac{xa}{2L^2}+\frac{x^2}{2L^2}\right)-\frac{2aL}{\pi\,x^2}F \\ \frac{4\,\gamma_s\,b\,h}{x}\left(1-\frac{x}{2b}-\frac{xb}{2h^2}+\frac{x^2}{2h^2}\right)-\frac{2bh}{\pi\,x^2}F \\ 2\,\gamma_s\,x\left(\frac{a}{L}+\frac{b}{h}-\frac{x}{L}-\frac{x}{h}+\frac{t}{x}\right)-2\gamma_b\,x-\frac{2t}{\pi\,x}F \end{bmatrix}$$

with

$$g_a = Ln\left[\frac{a}{x}\left(1+\sqrt{1-\left(\frac{x}{a}\right)^2}\right)\right]-\frac{L}{a}$$

$$t = \frac{1}{x^2}\left[\frac{V_o}{\pi}-\left(a^2\,L - \frac{1}{3}\,L^3\right)-\left(b^2\,h - \frac{1}{3}\,h^3\right)\right]$$

$$g_b = Ln\left[\frac{b}{x}\left(1+\sqrt{1-\left(\frac{x}{b}\right)^2}\right)\right]-\frac{h}{b}$$

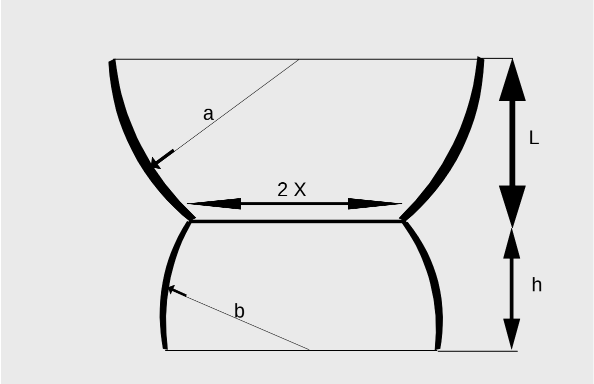
Numerical Procedures

- initial conditions $\longrightarrow$ $a = a_o$ $\quad b = b_o$ $\quad$ and $\quad x = 0$

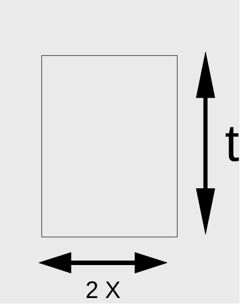
- Runge Kutta

Case 1: very small value of x, instability numerical ⇨ asymptotic approach

Case 2: elimination of the circular disc



Case 3:elimination of the spherical surfaces



Case 1: very small values for the magnitude of x

• in the limit where x /a , x /b and t /x are much smalle than 1 and higher order terms in $[k]\{\dot{\delta}\} = \{f\}$ are neglected

$$\dot{x} = \frac{\left[4\, D_b\, g_a\, g_b + D_s(g_a + g_b)\right](2\gamma_s - \gamma_b)}{g_a g_b x t^2} - \frac{4 D_b\, F}{\pi\, x^3\, t} \qquad \oplus$$

$$\dot{a} = -\frac{D_s(2\,\gamma_s - \gamma_b)}{g_a\, t\, a^2} + \frac{t\, D_b\, F}{6\,\pi\, x^3\, a^2\, g_a} \qquad \dot{b} = -\frac{D_s(2\,\gamma_s - \gamma_b)}{g_b\, t\, b^2} + \frac{t\, D_b\, F}{6\,\pi\, x^3\, b^2\, g_b} \qquad t = \frac{1}{x^2}\left[\frac{V_o}{\pi} - \frac{2}{3}\left(a^3 + b^3\right) + \frac{x^4}{4}\left(\frac{1}{a} + \frac{1}{b}\right)\right]$$

Of $\oplus$ with $1/g_a$ and $1/g_b$ negligible is obtained :

generalization of Coble's

$$x^6 = 96\, D_b \left[\frac{4(2\gamma_s - \gamma_b)}{\left(1/a_o + 1/b_o\right)^2} - \frac{F}{\pi\left(1/a_o + 1/b_o\right)}\right] Te$$

are used until :

$$\frac{x}{a} = 0.01$$

$Te \rightarrow$ *time* elapsed

Results

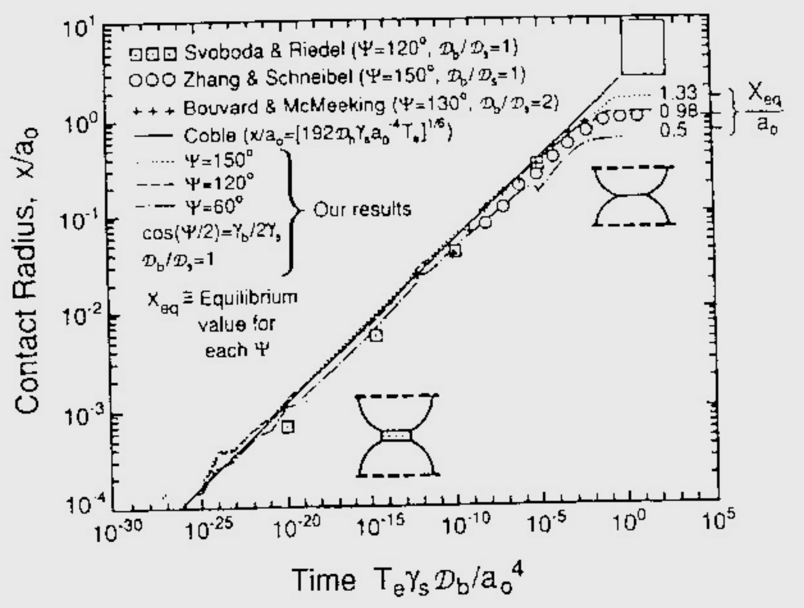
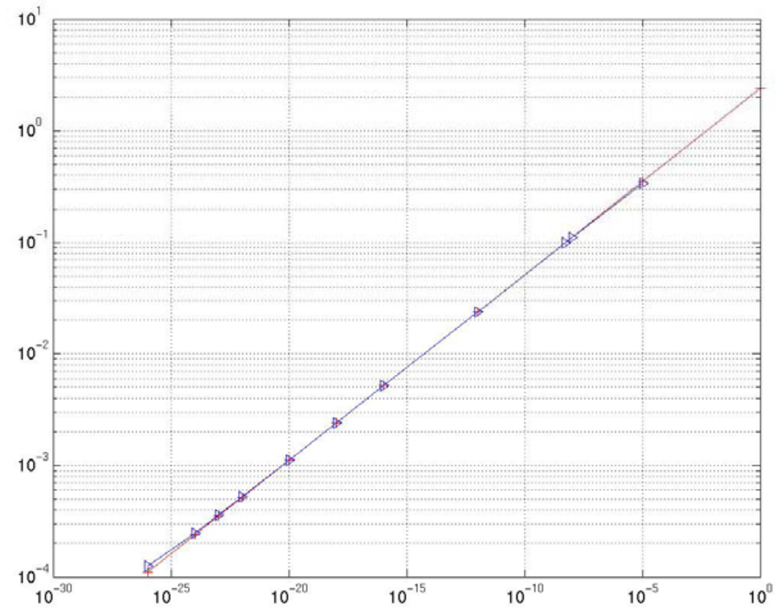


Fig. 3. Contact radius vs. time for the free sintering of a row of identical particles for various dihedral angles.

paper



numerical matlab

Results

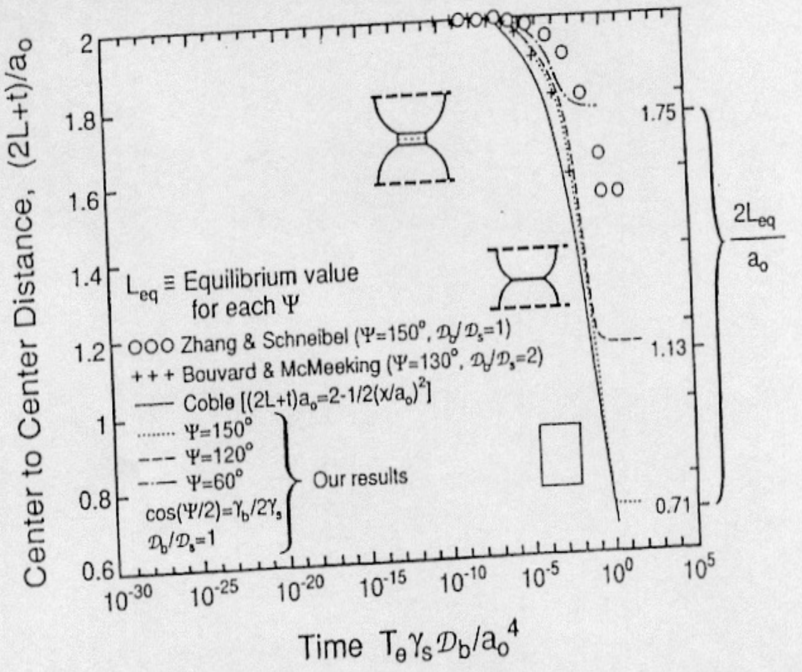


Fig. 4. Center-to-center distance vs. time for the free sintering of a row of particles for various dihedral angles.

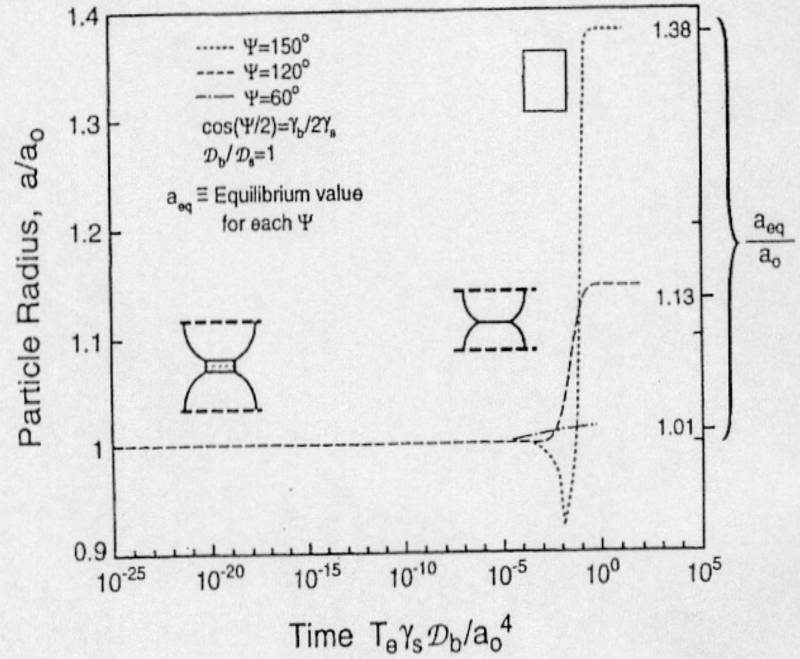


Fig. 5. Particle radius vs. time for the free sintering of a row of identical particles for various dihedral angles.