RHEOLOGY AND MIXING OF SUSPENSION AND PASTES

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PLAN

1- Rheology and Reactors
   Reactor performance problems caused by rheological behaviors of suspensions et pastes

2- Rheology of complex fluids
   Definition
   Classification of mixtures
   Non-Newtonian behaviors
   Behavior laws of viscoplastic fluids
   Thixotropy
   Viscosity equations
   Rheological measurements

3- Factors influencing the rheological behavior of fluids

4- Mixing of pastes in agitated vessels
   Agitator and utilization
   Geometric parameters
   Dimensional numbers
   Dimensionless numbers
1- Rheology and Reactor

Mass balance:

\[
\left( \frac{A_{j,\text{in}}}{\text{Flux}} \right) + \left( \frac{A_j}{\text{Production}} \right) = \left( \frac{A_{j,\text{out}}}{\text{Flux}} \right) + \left( \frac{A_j}{\text{Accumulation}} \right)
\]
DIMENSIONS OF REACTOR
IN VIEW OF SCALE CHANGE

PERFORMANCE OF REACTOR:

Thermodynamic and kinetic
of the reaction

Hydrodynamic

Operating parameters:
Nature of reagents
Pressure, temperature
Concentrations
Flow
Residence time

Composition
Conversion rate
RTD
Output

Mass and heat transfer

Geometric of reactor

SIMILARITY PRINCIPLE:

☐ Geometric similitude
☐ Kinematic similitude
☐ Energetic similitude
☐ Thermal similitude
ENCOUNTERED PROBLEMS WITH REACTOR

◆ Existence of dead matter and recirculation:

![Diagram showing stagnant fluid and recirculation]

◆ Presence of preferred passages

![Diagram showing preferred passages]

◆ OBJECTIVE:
Correct the flows or take it into consideration while designing the reactor
Ribbon impellers (agitators) for mixing Complex fluids
anchor  Helicoidal ribbon  Archemedian ribbon impeller

(2) mobiles couplés  (B) mobiles découpés
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2- Rheology of complex fluids

Model of flowing fluid between 2 plates in which one is mobile (upper plate) and the other is motionless (lower plate)

\[
\tau_{xy} = \frac{F}{S} = \eta \frac{\partial V_x}{\partial y} = \eta \dot{\gamma}
\]

- \(\tau_{xy}\) Shear stress
- \(\dot{\gamma}\) Shear rate
- \(\eta\) Dynamic or absolute viscosity coefficient

This rheological equation depends on the nature of the fluid and external conditions (T et P)

\[
\tau = \eta_a \dot{\gamma}
\]

- \(n\): Behavior index
- \(\eta_a\) Apparent viscosity
Characterization of the rheological behavior of fluids using rheograms:

- Graph representing the shear stress vs the shear rate

\[ \tau - \gamma \]

- Graph representing the shear stress vs the deformation

\[ \tau - \gamma \]

1- Newtonian behavior

\[ \tau = \eta_a \gamma \quad n = 1, \eta_a = \eta \]
Viscosity laws

Several models are available in literature including those for:

1- Homogenous fluids

- Carreau’s model
To define the characteristic time of the media

\[
\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[ 1 + \left( t_B \gamma \right)^2 \right]^{\frac{n-1}{2}}
\]

with \( \eta = \eta_\infty, \gamma \to \infty \)
\( \eta = \eta_0, \gamma \to 0 \)
\( t_B, \text{ Characteristic time} \)

- Ellis’ model

\[
\frac{\eta_0}{\eta} = 1 + \left( \frac{\tau}{\tau_{1/2}} \right)^{\alpha-1}
\]

\( \tau_{1/2} : \text{ Shear stress for } \eta = \frac{\eta_0}{2} \)
\( \alpha : \text{ Ellis’ parameters that depends to the behavior index} \)
2- Biphasic Fluids

Examples: Suspensions, pastes

When the proportion of the solid is taken into account through the volume fraction $\phi$:

\[ \phi = \frac{x}{\frac{x}{\rho_s} + \frac{1-x}{\rho_1}} \]

$X$: the concentration of solid

$\rho_s$: density of the solid phase

$\rho_1$: density of the liquid phase

For a dense and random packing of particles in the liquid phase:

$\phi_{\text{max}} = 0.64$
**Behavior Laws:**

- For diluted suspensions of spherical particles:

\[ \eta = \eta_l (1 + 2.5\phi) \]  
  Einstein’s Law

- For high values of \( \phi \):

\[ \eta = \eta_l \left(1 + \frac{\phi}{\phi_m}\right)^{-q} \]  
  Krieger-Dougherty’s Law

\[
\eta = \eta_l \left[1 + 0.75 \frac{\phi}{\phi_{\text{max}}} \left(1 - \frac{\phi}{\phi_{\text{max}}}\right)^{B}\right]
\]

Loi Chong et al.  
With \( B=2 \)

If \( B \) is not a constant:

\[
\eta = \eta_l \left[1 + A \frac{\phi}{\phi_{\text{max}}} \left(1 - \frac{\phi}{\phi_{\text{max}}}\right)^{B(\tau,\phi)}\right]
\]

Nzihou et al.
2- Non-Newtonian behaviors

2.1- Viscous behavior

\[ \tau = \eta_a \gamma \]  
Ostwald De Waele’s Law

- \( n = 1 \), Newtonian fluid
- \( n < 1 \), Shear thinning fluid
- \( n > 1 \), Shear thickening fluid

2.2- Viscoelastic behavior

\[ \tau = \tau_0 + \eta_a \gamma \]  
Hershell-Bulkley’s Law

- \( \tau_0 \): Yield stress
- \( n = 1 \), Bingham’s fluid and \( \eta_a = \eta_B \)
- \( \eta_B \): Bingham’s plastic viscosity
For a number of food and cosmetic fluids:

\[ \sqrt{\tau} = \sqrt{\tau_0} + \sqrt{\eta_c} \quad \text{Casson’s Law} \]

\( \eta_c \) : Casson’s plastic viscosity
Manifestation of different behaviors
3- Thixotropic Behavior

Time effect in non-newtonian fluid
This is a reversible process.

Origin of the behavior: Breakdown, equilibrium, rebuilding
Representation of the rheological behavior of thixotropic fluids

Behavior of law:

\[ \tau = (\tau_0 + \lambda_c \tau_s) + \eta_B \dot{\gamma} \]

\( \tau_s \): Structure stress

\( \lambda_c = 1 \) Structure at rest

\( \lambda_c = 0 \) Complete breakdown of the structure at a high shear rate

Formation rate breakdown of the thixotropic structure:

\[ \frac{d\lambda_c}{dt} = a(1 - \lambda_c) - b\lambda_c \dot{\gamma} \]

a and b are specific parameters of the mixture. They must be determined experimentally.
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3- Factors influencing the rheological properties of mixtures

4- Mixing of pastes in agitated vessels
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   Dimensionless numbers
3- Important factors influencing the rheological behavior

To be discussed during the course

- Density and volume fraction of solids
- Porosity of the solid
- Particle size distribution
- Form/Shape of particles
- Surface area
- Interfacial properties (chemical composition and structure)
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4.2 Geometric parameters

H/D, nR, I/d or I/D, d/D, p/d or p/D, h_a/d, e/D, w/d, Y/D, L, n_p

d, diameter of the agitator (m)
D, interior diameter of the reactor (m)
e, gap (m)
H, height of the suspension in the reactor (m)
ha, total height of the the agitator (m)
I, width of ribbon (m)
L, length of blades (m)
nR, number of ribbon
np, number of blades
p, helix gap

Example: Archimedean screw impeller

d/D=0,95
0,5< p/d <2
0,044< I/D <0,33
40< Re_e <270
0,023< e/D <0,097
n_R = 1
4.3 Dimensionless numbers

The Reynolds number in effect in the agitation

Representation of the flow regime (the inertia and viscous effect)

\[ \text{Re}_e = \frac{\rho N d^2}{\mu_e} \]

N, rotation speed (s\(^{-1}\)), d agitator’s diameter (m), 
\( \mu_e \) effective viscosity of suspension (Pa.s) obtained with the rheometer, \( \rho \) density of the suspension in kg/m\(^3\)

Flow regimes:

\( \text{Re}_e < 10^{-50} \) : laminar flow
\( 10^{-50} < \text{Re}_e < 10^4 \) intermediate flow
\( \text{Re}_e > 10^4 \) turbulent flow
4.4 Dimensional number

Agitation power

Necessary driving force for the agitator

\[ P = K_p \mu e N^2 d^3 \]

Calculation of \( K_p \), constant of helical mobile.
A few equations:

\[ k_p = 66 n_r (p/d)^{-0.73} (e/d)^{-0.6} (I/d)^{0.5} (H/d) \]  
Hall’s correlation

\[ k_p = 52.5 n_r^{0.5} (p/d)^{-0.5} (e/d)^{-0.5} \]  
Nagata’s correlation

\[ k_p = a_M (p/d)^{0.7} (I/d)^{-0.03} \text{Re}^{b_M} \]  
Archimedean screw ribbon
Effective shear rate

\[ \dot{\gamma}_e = K_s N \]

\( K_s \), Metzner and Otto’s constant given by the following correlations.

A few correlations:

For 0.026 < \( \frac{e}{D} \) < 0.16 \( K_s = 34 \, -114 \left( \frac{e}{d} \right) \) Shamlou’s correlation

\[ K_s = 8.9 \left( \frac{e}{D} \right)^{-1/3} \] Kuriyama’s correlation

\[ k_s = 25 \left( \frac{d}{D} \right)^{0.5} \left[ \frac{\left( \frac{p}{d} \right)}{\pi^2 + \left( \frac{p^2}{d^2} \right)} \right]^{-0.5}^{-0.15} \] Bakker’s correlation

If 0.023 < \( \frac{e}{D} \) > 0.097 ; 0.91 > \( \frac{p}{D} \) < 1.9 ; 0.077 < \( \frac{I}{D} \) < 0.2, we have:

\[ k_s = 38.3 \left( 0.814 \right)^{1/n} \left( \frac{p}{d} \right)^{-0.14} \left( \frac{I}{d} \right)^{-0.024} \] Yap’s correlation

with \( n \), the behavior index determined by the rheometer
Mixing time

\[ N = \frac{N t_M}{t_M} \]

To be discussed during the course (see the graph)

1. ancre \((d/D = 0.98)\) - fluide newtonien
2. vis hélicoïdale \((d/D = 0.62)\) sans tube de tirage - fluide newtonien
3. vis hélicoïdale \((d/D = 0.62)\) avec tube de tirage - fluide newtonien

Influence de la rhéologie du fluide avec le même ruban hélicoïdal

4. ruban hélicoïdal - fluide newtonien
5. ruban hélicoïdal - fluide rhéofuidifiant
6. ruban hélicoïdal - fluide viscoélastique
4.5 Heat transfer (Nusselt Number)

It is necessary to clear the flux of reactive heat as well as the heat generated by the agitator which can reach several kW/m³ for suspensions having an high effective viscosity. The heat-exchange surface coefficient and therefore Nusselt’s equation, Prandl’s equation, the dissipated heat, the volumic capacity of cooling, the exchange coefficient agitated-wall suspension can be determined.

Heat transfer

Dimensionless numbers: Nusselt’s equation, Prandtl’s equation, Reynolds’ equation, are defined in order to establish the relation between different system variables and the importance of certain phenomenon in relation to others.
- Nu corresponds to the relationship between the transport of heat by conduction-convection and the transport of heat by conduction

\[ Nu = \frac{hD}{\lambda} \]

- Prandtl’s equation represents the relationship between the molecular diffusivity of the matter and the molecular diffusivity of heat.

\[ Pr = \frac{C_p \mu}{\lambda} \]

- Reynolds’ equation represents the relationship between inertial forces and viscous forces.

\[ Re = \frac{\rho u D}{\mu} \]

- heat transfer in Newtonian fluids

\[ Nu = B(\text{géométrie}) Re^x Pr^y \left( \frac{\mu}{\mu_p} \right)^z \]

- heat transfer in Non-Newtonian fluids

\[ Re_{a.eq} = \frac{\rho N d^2}{K \gamma_{eq}^{n'} - 1} = \frac{\rho N d^2}{K (K_s N)^{n'} - 1} = \frac{\rho N^{2-n'} d^2}{K.K_s^{n'-1}} \]

The viscosity has been replaced by the equivalent viscosity (determined by the Metzner-Otto method).
Examples of correlations:

<table>
<thead>
<tr>
<th>Fluide</th>
<th>D</th>
<th>D/d</th>
<th>p/d</th>
<th>ha/d</th>
<th>Nu, RF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nagata [1]</td>
<td>0.30</td>
<td>1.05 - 1.25</td>
<td>1</td>
<td>1</td>
<td>N, RF</td>
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<tr>
<td></td>
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<td></td>
<td>(Nu = 1.39 \frac{Re}{D}^{1/3} Pr^{1/3} \frac{Vis}{D}^{0.12} (\pi/d)^{-1.3} )</td>
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<td></td>
<td></td>
<td></td>
<td>(1 &lt; Re &lt; 1000 )</td>
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<tr>
<td>Mitsuishi [36] (2)</td>
<td>0.40</td>
<td>1.053</td>
<td>1</td>
<td>1.6</td>
<td>N, RF</td>
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<td></td>
<td></td>
<td>(Nu = 0.78 \frac{Re}{D}^{1/3} Pr^{1/3} \frac{Vis}{D}^{0.18} )</td>
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<td>(1.5 &lt; Re &lt; 10 )</td>
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<td></td>
<td>(Nu = 0.53 \frac{Re}{D}^{1/2} Pr^{1/3} \frac{Vis}{D}^{0.14} )</td>
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<td>(10 &lt; Re &lt; 180 )</td>
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<td></td>
<td>(Nu = 0.22 \frac{Re}{D}^{1/2} Pr^{1/3} \frac{Vis}{D}^{0.14} )</td>
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<td>(180 &lt; Re &lt; 1000 )</td>
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<tr>
<td>Shamiou [8]</td>
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<td>N, RF</td>
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<td></td>
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<td></td>
<td></td>
<td>(Nu = 0.17 \frac{Re}{D}^{0.16} Pr^{1/3} \frac{Vis}{D}^{0.19} \frac{Re}{\rho}^{0.12} (\pi/d)^{-0.42} (\pi/D)^{-0.24} )</td>
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<td>(Re &lt; Re &lt; 1 )</td>
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<td></td>
<td>(Nu = 0.45 \frac{Re}{D}^{0.5} Pr^{1/3} \frac{Vis}{D}^{0.14} )</td>
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<td></td>
<td>(10 &lt; Re &lt; 1000 ) at (Re &gt; Re )</td>
</tr>
</tbody>
</table>

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SOME BIBLIOGRAPHICAL REFERENCES


H. Desplanches et J.L. Chevalier, Les Techniques de l’ingénieur (à partir de J 3 800 – 1)


Mezaki et al., Engineering data on mixing